

Fig. 6 Frames from a high-speed movie of shot 27 as the silver projectile accelerates through the transparent tube with luminous combustion established on the afterbody (projectile traveling right to left, frames progressing top to bottom).

### Conclusions

Flow visualization techniques for transient and steady combustion in the ram accelerator have been demonstrated. Flow visualizations indicate intense combustion around the projectile until the obturator is far downstream of the projectile; stable combustion is then established on the projectile mid- and aft-sections. These observations suggest that reduction in the number of unstarts, as well as potential performance increases, may be achieved by control of the obturator location relative to the projectile during the starting process.

### Acknowledgments

A. Horst, T. Minor, and D. Kooker have supported both the experimental and numerical simulation aspects for the HIRAM project since its inception and have contributed to this work through various technical discussions. C. Ruth, A. Koszoru, J. Hewitt, and J. Tuerk provided support in the experimental and photographic aspects of the project.

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## Study on Thermal Strain Using Subscale Specimens

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### Introduction

A NONLINEAR relationship between hoop strains and temperatures is often observed in strain evaluation cylinder (SEC) experiments. This phenomenon is not consistent with the simplified equation mentioned in a conventional manual<sup>1</sup> and recently applied in round-robin analysis<sup>2</sup> for assessing stress-free temperature and thermal expansion coefficient of propellant. Moreover, the traditional measure techniques, x-ray and intermicrometer, have some drawbacks from the experimental point of view.

This work elucidates a modified equation-resolving experimental disagreement with the simplified equation, describes a technique improving drawbacks of measurement, and discusses finite element analysis (FEA) results pointing to displacement rather than strain for comparison between analysis and the SEC experiment in the geometric nonlinear case.

### Theory Consideration

#### Linear Solution

According to the ICRPG manual,<sup>1</sup> a simplified solution for SEC data analysis is defined as follows:

$$\varepsilon = -(p/2)(\lambda^2 - 1)(\beta T + l) \quad (1)$$

in which  $\varepsilon$ ,  $p$ ,  $\lambda$ ,  $\beta$ ,  $T$ , and  $l$  are, respectively, the hoop strain, Parr factor, grain's external diameter/mandrel diameter, volume thermal expansion coefficient, temperature, and constant. Equation (1) shows that the relationship between hoop strains and temperatures is linear, which is not consistent with the report depicting a nonlinear phenomena.<sup>2</sup> Having considered the deformed configuration of propellant grain, a modified solution can be derived as follows:

$$\varepsilon = -(p/2)\{[dm/d(T)]^2\lambda^2 - 1\}(\beta T + l) \quad (2)$$

in which  $dm$  is the mandrel diameter and  $d(T)$  is the inner-bore diameter of propellant grain at different temperatures. Thus, Eq. (2) indicates a nonlinear relationship of hoop strains vs temperatures and leads to a convex shape which is consistent with the data of the report.<sup>2</sup>

Received Dec. 31, 1993; revision received Oct. 14, 1994; accepted for publication May 16, 1995. Copyright © 1995 by the American Institute of Aeronautics and Astronautics, Inc. All rights reserved.

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Even though Eq. (2) can more properly describe the variation of hoop strain vs temperature, there is still a limitation for large deformation owing to the assumption of a small strain in deriving modified solution. For a large deformation, FEA is an effective method to give a calculated solution for grain structural analysis.

#### FEA Method

1) Computer code: A general-purpose FEA code, ABAQUS, is applied in the study.

2) Element mesh: The geometry of SEC is reduced to plane strain and eight node biquadratic elements.

3) Loading condition: The 80% web fraction (WF) SEC is under a thermal shock loading from +60 to -40°C in 10 s.

4) Case properties: The properties of SEC case are adopted as follows:  $E_c = 20,386 \text{ kg/mm}^2$ ,  $\nu_c = 0.32$ ,  $Cp_c = 0.2865 \text{ cal/g } ^\circ\text{C}$ ,  $k_c = 29.76 \text{ kcal/m h } ^\circ\text{C}$ , and  $\alpha_c = 1.07\text{E-}5 \text{ 1/}^\circ\text{C}$ . However, the corresponding properties of propellant are measured by experiments and described in the next paragraph.

## Experiments

#### SEC Specimen

The SEC case is made by AISI 316 steel with 5 mm thickness and 60 mm o.d. The ratio of i.d. to length for the case is 1:7. The HTPB/AP/Al/RDX propellant is cast into 80, 75, 70, and 65% WF SECs and cured at 65°C. The mandrel diameters for 80, 75, 70, and 65% WF SECs are 10, 12.5, 15, and 17.5 mm, respectively.

#### Diameter Measurements

An on-line system as shown in Fig. 1 was established to improve the technique of measuring SEC diameter. This on-line system includes a personal computer, scan controller, temperature scanner, thermocouple, strain gauge scanner, and a transducer, which were depicted in authors' work.<sup>3</sup> The transducer is mounted at the bore center of SEC and subsequently connected to the scan controller and personal computer through the strain gauge scanner. A thermocouple mounted at the inner-bore surface of SEC and connected to the temperature scanner is used to detect the temperature of SEC inner-bore surface. After the connections, the system can automatically record the real-time variations of diameters and temperatures while SECs are in thermal loading condition.

Intermicrometer was used to measure the SECs' inner-bore diameters to verify the accuracy of measurement by the on-line system.

#### Propellant Properties

##### Expansion Coefficient $\alpha$

The linear thermal expansion coefficient of the propellant, averaged  $9.85 \pm 0.72\text{E-}5 \text{ 1/}^\circ\text{C}$  from +60 to -60°C, was measured by thermomechanical analyzer (TMA), Perkin-Elmer TMS-2, with a temperature rate of 5°C/min.

##### Specific Heat $C_p$

The specific heat of the propellant, averaged  $0.264 \pm 0.0014 \text{ cal/g } ^\circ\text{C}$  from +60 to -60°C, was measured by differential scanning calorimetry (DSC), Perkin-Elmer DSC-2C, with a temperature rate of 5°C/min.

##### Heat Conductivity $k$

The heat conductivity of the propellant, varying in the -10 to +70°C range from 0.432 to 0.381 kcal/m h °C, was measured by QTMD2, constructed by Kyoto Electronics Manufacturing Co., LTD.

##### Propellant Modules $E$

1% relaxation modulus of the propellant at different temperature were measured by a Shimadzu A universal test ma-

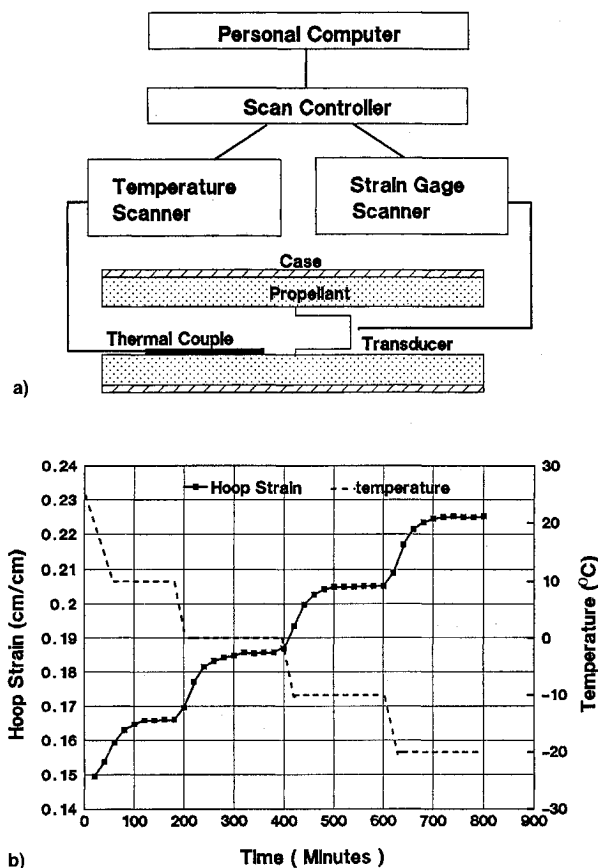


Fig. 1 On-line measuring system: a) system sketch and b) measuring result.

chine. All data are shifted to construct a master curve of  $\log E$  vs  $\log a_T$  and subsequently transferred to Prony series for use in ABAQUS. Relaxation time is  $t$  and  $a_T$  is the shift factor of the master curve.

## Results and Discussion

#### SEC Experiments

According to the ICRPG manual, x-ray and intermicrometer are suggested methods to measure SEC's inner-bore diameter. From the experimental point of view, there are three drawbacks that may affect the accuracy of measurement. First, temperature is difficult to be controlled during measurement when SEC is taken out of the oven. Second, it is very hard to assess the diameter by examining the ambiguous interface between propellant and case in the film of x-ray. Third, too much force applied to intermicrometer may damage the inner-bore surface of SEC grain. Our on-line system with real-time measurement ability avoids these drawbacks.

The inner-bore diameters of SECs measured by the on-line system and intermicrometer are listed in Table 1. The data for 65 and 75% WF SECs measured by the system are very close to those measured by intermicrometer. The on-line system can measure 16 SECs at the same time because both the strain gauge scanner and temperature scanner have 16 channels.

Moreover, Fig. 1, a result of real-time measurement in the slow-cooling experiment, indicates that the time for hoop strain reaching steady state is not more than 1 h. The total time needed for the measurement from 25 to -20°C is only about 0.6 days (800 min). According to ICRPG manual, the same experiment needed about six days. The comparison reveals that the on-line system can conserve experimental time for being sure that temperature equilibrium is absolutely reached in the grain.

Table 1 Inner-bore diameters measured with the instrument and intermicrometer

Temperature, °C	Inner diameters of different WF, mm					
	75%			70%		65%
	(a)	(b)	(c)	(a)	(a)	(b) (c)
10	13.7260	13.8137	0.63	16.0454	18.4336	—
0	13.9232	13.9598	0.26	16.1962	18.5625	18.5373
-10	14.0619	14.1458	0.59	16.3436	18.6975	18.7227
-20	14.2101	14.3396	0.90	16.4837	18.8430	18.8742
-30	14.3039	14.4578	1.06	16.6430	18.9654	18.9617

(a) Data measured by the on-line system.

(b) Data measured by intermicrometer.

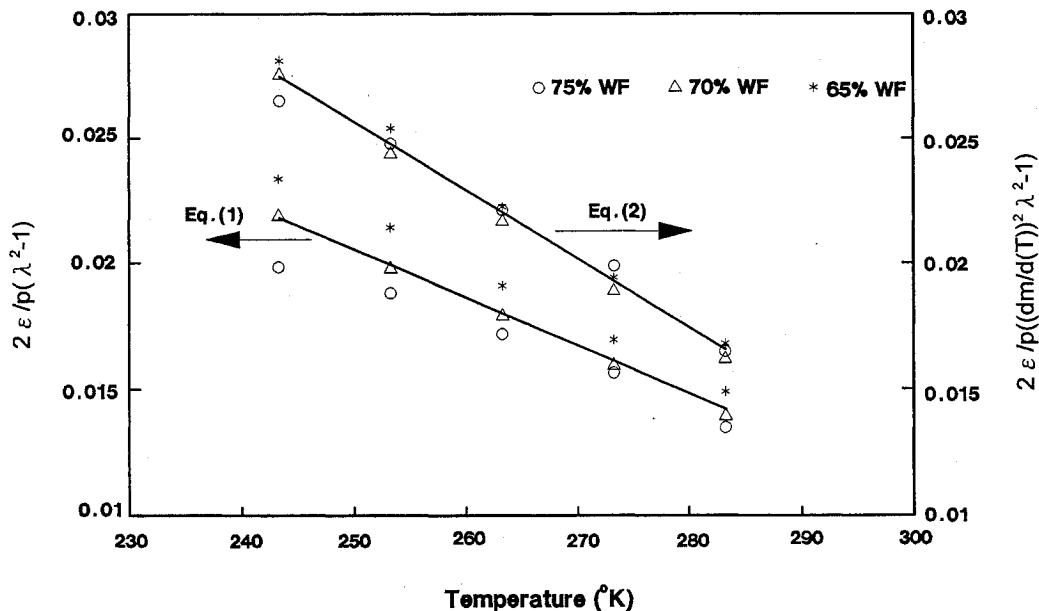
(c) Deviation (%) =  $|(a) - (b)| * 100/(b)$ .

Fig. 2 Linear regression analysis in accordance with Eqs. (1) and (2).

### Data Treatments

According to data procedures of the ICRPG manual and our SEC data (Table 1), the Poisson's ratio of the HTPB propellant is verified around 0.499–0.5. Therefore, the assumption of Poisson's ratio near 0.5 for Eqs. (1) and (2) is satisfied.

Figure 2 illustrates that the data points of 65 and 75% WF SECs for Eq. (1) regularly deviate from the regression line. This deviation reveals that the three slopes, corresponding to three thermal expansions of the three SECs, are obtainable if the data are separately calculated in accordance with WF. In other words, the thermal expansion coefficient of the propellant is not unique and depends on the geometry of propellant grain. This conclusion is unreasonable because the thermal expansion coefficient measured by TMA is independent of the geometry of propellant grain and mostly constant in narrow range of testing temperatures.

Figure 2 also depicts that the data points are close to the regression line derived from Eq. (2) rather than from Eq. (1). Compared with the results from Eqs. (1) and (2) in Table 2, the linear thermal expansion coefficient measured by TMA is  $9.85\text{E-}5\text{ }1/^{\circ}\text{C}$ , which is almost the same to  $9.13\text{E-}5\text{ }1/^{\circ}\text{C}$  resulting from Eq. (2) rather than  $6.34\text{E-}5\text{ }1/^{\circ}\text{C}$  resulting from Eq. (1). In addition, the statistic parameter  $R^2$  for Eq. (2) is 0.99, which is greater than 0.89 for Eq. (1). These reveal that Eq. (2) is better than Eq. (1) to obtain a reasonable stress-free temperature and to describe the hoop-strain behavior of SECs.

Recently, Eq. (1) was applied in round-robin analysis leading to two stress-free temperatures and two thermal expansion coefficients for PBAN propellant used in Minutemen missiles.<sup>2</sup> This result is not consistent with the nature of solid propellant

itself. Moreover, the relationship between hoop strains vs temperatures is in a convex shape, which is consistent with the solution of Eq. (2).

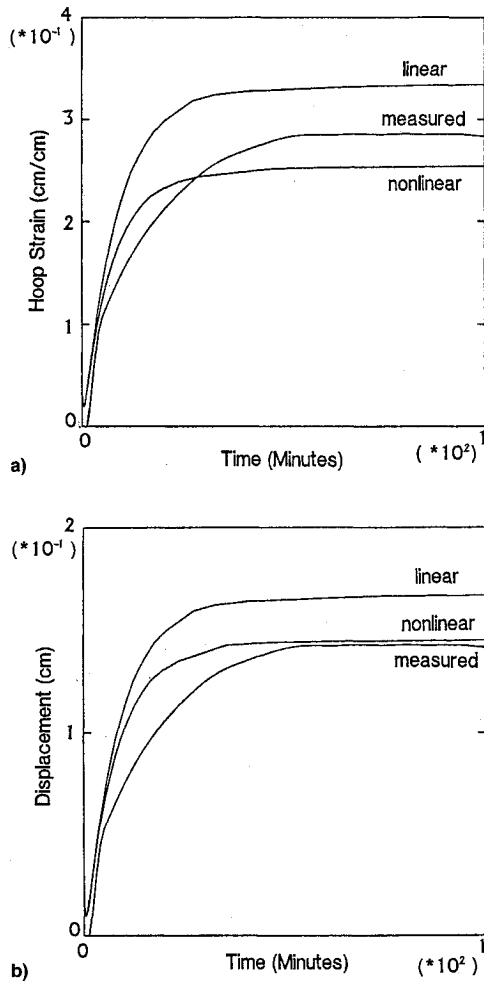
### FEA Results

Based on the properties of the SEC case and the propellant mentioned previously, FEA results for 80% WF SEC in thermal shock can be calculated as shown in Fig. 3. In linear FEA, both hoop strain  $\{[d(T) - dm]/dm\}$  and displacement  $[d(T) - dm]$  deviate 18.5% from SEC experiments. In nonlinear FEA, hoop strain and displacement deviate 10 and 2.8%, respectively, from experiments. The different deviations of hoop strain and displacement in the nonlinear case are due to the fact that hoop strains of the SEC experiment are calculated from  $[d(T) - dm]/dm$ , which is based on the assumption of small deformation. In fact, the data variations of diameters rather than those of hoop strains are measured in an SEC experiment. Owing to that, the displacement is recommended to be a parameter to compare nonlinear analysis and experiment. In addition, only geometric nonlinearity is considered in the FEA, showing great agreement with the experiment even though other factors could result in experimental disagreement with FEA such as dilatation and nonconstant thermal expansion coefficient. This reveals that geometric nonlinearity is an important factor for FEA of propellant grain loaded by high strain.

Mentioned in the report of round-robin analysis,<sup>2</sup> the analysis panel recommended that reliable thermomechanical nonlinear constitutive models (including finite deformation effects, nonconstant thermal expansion, and dilatation behavior) need

**Table 2** Data comparison between Eqs. (1) and (2)

Equation	$R^2$	$\beta$ , $1/^\circ\text{C}$	$\alpha$ , $1/^\circ\text{C}$	$T_s$ , $^\circ\text{C}$
(1)	0.89	1.90E-4	6.34E-5	85
(2)	0.99	2.74E-4	9.13E-5	71

**Fig. 3** Comparison between FEA and experiments in thermal shock condition: a) hoop strains and b) displacements.

be continuously developed. According to the above discussion, finite deformation could be a dominant effect for constructing thermomechanical nonlinear models.

### Conclusions

Our on-line system is workable to substitute x-ray and interferometer for measuring the i.d. of SEC.

A modified equation is derived for accurately assessing a stress-free temperature and thermal expansion coefficient of solid propellant grain.

For finite deformation, displacement is a proper parameter for comparing the SEC experiment and FEA.

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## Development of Equations of State for Compressible Liquids

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### Nomenclature

- $a$  = speed of sound
- $c_p$  = constant pressure specific heat
- $c_v$  = constant volume specific heat
- $e$  = internal energy
- $L$  = length
- $M$  = Mach number
- $\bar{P}$  = reference pressure
- $p$  = pressure
- $R$  = universal gas constant
- $s$  = entropy
- $T$  = temperature
- $\bar{T}$  = reference temperature
- $u$  = fluid velocity
- $\alpha$  = cubical expansion coefficient
- $\beta$  = isothermal bulk modulus
- $\kappa$  = reference density
- $v$  = volume
- $\rho$  = density

### I. Introduction

IN this Note two novel, general, caloric, and thermal equations of state (EOS) for liquids are derived from first principles. These equations are thermodynamically compatible. Moreover, an expression for entropy consistent with the new EOS is also developed. Curve-fitting techniques can be used to adjust physically significant equation parameters to fit the EOS to available thermodynamic data for particular liquids. The EOSs and the entropy expression are used in conjunction with the equations of motion to model liquid fluid flow. Specifically, they have been used in propulsion system dynamic modeling where the EOS have been successfully tailored to liquid hydrogen ( $\text{LH}_2$ ).<sup>1</sup> The derived model is compared to National Bureau of Standards (NBS) data with good results.

The basis for this work is rooted in the fact that the conservation equations of continuity, momentum, and energy (CME) are employed to describe the dynamic behavior of a fluid. In order to link the kinetic behavior of the medium with its thermodynamic behavior, the EOS are used. Together with the EOS, the CME equations hold kinetic and thermodynamic information to describe fluid system dynamics. The interest lies in finding suitable thermal and caloric EOS as density and internal energy appear in the CME equations. The EOS, cast in usual fluid dynamic variables of pressure and temperature, prove useful in the derivation of the pressure expression. The CME system can then be reformulated in terms of pressure, velocity, and entropy. Such a configuration readily provides insight when considering certain processes, e.g., isentropic. The CME equations for an ideal gas system have already been formulated in terms of entropy, pressure, and velocity.<sup>2</sup>

Traditionally in fluid dynamic problems, a liquid is considered to be incompressible. This approximation affords significant simplification to the governing CME equations and it

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